Theoretical

Simulating Large Social Networks In Agent-Based Models: A Social Circle Model

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Many social simulations require an underlying model of social networks. However, none of the standard network models fits well with sociological observations of real social networks. Taking the idea of social circles, this paper presents a simple system to create large networks in agent-based models that incorporate key aspects of large social networks such as the differing sizes of personal networks, high clustering, positive assortativity of degree of connectivity, and low density. This social circle model is very flexible and can be used to create a wide variety of artificial social worlds.

Introduction

The sociological literature describes the typical features of social networks, yet none of the four basic models of networks—regular, random, small-world and preferential attachment—adequately reproduce those features. This paper introduces a new approach for modelling large social networks in agent-based models, one that does reproduce the features of real social networks. It expands on an idea originally described in Hamill & Gilbert (2009) and further developed in Hamill (2010).

Basic Characteristics Of Social Networks

Personal (or ego-centric) networks are representations of the relationships between one person and others: friends, family, acquaintances, work colleagues and so on. A social network is an aggregation of personal networks. The size of a personal network—in network jargon, called its degree of connectivity—will depend on what relationships are used to draw up the network. Following Boissevain (1974: 47-8), and drawing on a range of research, five groups can be broadly defined according to the strength of ties:

- Strongest: closest relatives and a few close friends; probably totalling about five;
- Strong: emotionally important very close friends and relatives with whom relationships are actively maintained; a further five to ten;
- Medium: emotionally important very close friends and relatives with whom

relations are passively maintained; another 20 to 30;

- Weak: people who are important for their "economic and social purposes and the logistics of everyday life"(ibid); some 100-150 more;
- Weakest: acquaintances, whose names may not be known: maybe adding over 500.

Howsoever defined, personal networks vary in size between individuals. A few people will be very well-connected (Boissevain, 1974: 124-5; Travers & Milgram, 1969) and many much less so. Data are, however, scarce: often researchers limit the number of names collected from their respondents and even when respondents are given full freedom, only summary statistics are reported. Nevertheless these often indicate a right-skewed, even fat-tailed, distribution with a few people having very large personal networks (e.g., Boase, 2008; Thelwall, 2008; Fischer, 1982: 38-9).

Not all social networks are fat-tailed, however. Bruggeman (2008: 34) pointed out that "the distribution of close friendships cannot have a fat-tail". As Aristotle (c300BC/1996, Book 9: x, 3-6) noted, "the number of one's friends must be limited" because, in modern terminology, the maintenance of social networks is not costless, resulting in cut-offs above a certain degree (Watts & Strogatz, 1998; Amaral *et al.*, 2000; Barthélemy, 2003). Thus, any model should constrain the size of personal networks because of the time and effort needed to maintain them. The model should also permit the size of personal networks to vary between individuals, with the possibility of some individuals having much larger personal networks than average.

Aristotle (c300BC/1996: Book 9: x, 3-6) also noted that "one's friends must also be friends of one another". More recently, Granovetter (1973) suggested that the stronger the ties the more similar people are. Indeed, homophily—the principle that contacts between similar people occur more often than among dissimilar people—is a key characteristic of social networks. McPherson *et al.* (2001) reported that, in the US, race and ethnicity are the most important factors followed, in order, by "age, religion, education, occupation and gender". In the analysis of social networks, the extent to which one's friends are also friends of each other is measured by the clustering coefficient, which tends to be between a quarter and a half, but can lie outside these ranges (e.g., Scott, 1991: 80-2; Fischer, 1982: 145).

The network density of a whole network is the ratio of the actual number of links to the total possible. If everyone knew everyone else, then the density would be equal to one. That may be the case in small communities, but is clearly not in larger ones. Even if on average an individual knows, in some sense, a few thousand people, that is only a tiny fraction of the almost 7 billion people on the planet. Thus global density is low.

Despite this low density, the 'small world effect'—that anyone in the world can be reached by a few steps—was first noted 80 years ago (Karithny, 1929/2006). In network terminology, the path length, the most direct route between any pair of individuals, is short (Watts, 2004: 38). Pool & Kochen (1978/9) argued, using a thought experiment, that if links were random, the small world phenomenon would rarely be observed, but when it was, the path length would be very short, with only two links; in contrast, they suggested that Americans were linked by just seven intermediaries due to the structure of society, which reflects the tendency of similar people to mix with other similar people, i.e., to cluster. While there is abundant anecdotal evidence supporting the small world effect, scientifically-based evidence is thin:

- Milgram's famous experiment suggested that there were "six degrees" of separation, although this is based on just 64 chains completed by middle-class Americans (Travers & Milgram, 1969). Watts (2004: 134) reported that "only a handful of other researchers had attempted to replicate Milgram's findings, and their results were even less compelling than his".
- On the basis of a study undertaken between 2001 and 2003 using email, Dodds *et al.* (2003) concluded that social searches can reach their targets in a median of five steps within countries and seven when the chain extends between countries. However, of the 24 thousand message chains initiated, just 384 or 1.6 percent were completed.

But as Watts (2004: 136) pointed out, just because people cannot find a short path does not mean that it does not exist. Leskovec & Horvitz (2007) overcame the search problem by analysing communication links between 1,000 people globally, and found an average path length of 6.6 but a maximum of 29. However, Liben-Nowell & Kleinberg's (2008) analysis of the progress of internet chain letters found that rather than spreading widely and reaching many people in a few steps, as the small-world model would suggest, they actually followed "a very deep, tree-like pattern, continuing for several hundred steps". Thus, as Dodds *et al*. (2003) reported, "much about this 'small world' hypothesis is poorly understood and empirically unsubstantiated".

Recently Newman (2003; Newman & Park, 2003; Newman *et al*., 2006: 555) proposed that a key feature of social networks that distinguishes them from other types of network is positive assortativity of the degree of connectivity, i.e., those with many links link to others with many links. He found that this is not the case for technological networks (such as power grids and the internet) and biological networks (such as food webs and neural networks) where assortativity was negative. Onnela *et al.* (2007) also found evidence of positive assortativity in their study of mobile phone use. Bruggeman (2008: 35) has suggested that positive assortativity is a type of homophily: sociable people like other sociable people. Although more work is needed to establish whether positive assortativity is always a feature of social networks, it seems that a social network model should, for now, display this feature.

The size, structure and membership of a personal network will change over time: "a person's network is a fluid, shifting concept" (Boissevain, 1974: 48). For example, Grossetti (2005) reported "a constant turnover" in personal relationships: developing from family at birth though to friends at school, adding co-workers and neighbours in adulthood. Key life stage events, such as marriage, affect both the size and structure of personal networks. Kin relationships are more likely than friendships to be maintained even if contact is infrequent and the social ties are weak (see, for example, Kalmijn & Vermunt, 2007; Wellman *et al*., 1997). Over ten years, around a quarter of close ties persist (Wellman *et al*., 1997; Suitor & Keeton, 1997). However, longitudinal studies are rare (McPherson *et al*., 2001). Nevertheless, it is clear that a good model of personal networks should allow them to change considerably over time.

To sum up, drawing on Bruggeman (2008: 36) and Wong *et al.* (2006), together with the above discussion, it appears that personal networks should:

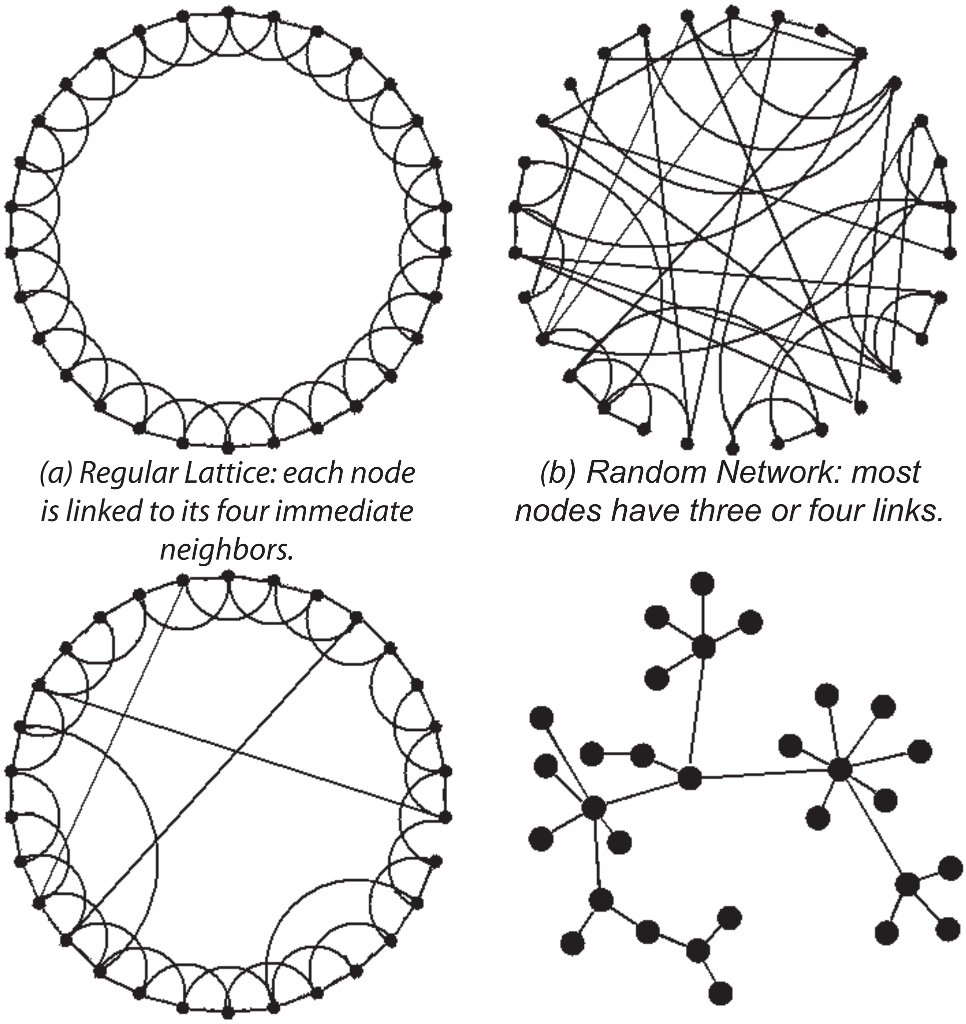
- Be of limited size, the limit depending on the type of relationships being studied;
- Vary between individuals, with a right-skewed distribution of degree of connectivity except for very close relationships;
- Display high clustering, i.e., members of an individual's personal network should tend to know each other to reflect homophily, and;
- Change over time.

And overall, a model of a social network should have:

- A low whole network density, i.e., only a very few of the potential links in the network should actually exist;
- Positive assortativity by degree of connectivity, i.e., those with large personal networks tend to know others with large personal networks;
- Communities, i.e., groups of people that are "highly connected within themselves but loosely connected to others" (Wong *et al*., 2006), and;
- Short path lengths, i.e., others can be reached in a small number of steps.

Four Basic Network Models

Four basic types of network model are commonly found in the literature and Figure 1 shows an example of each. The regular lattice, shown in Figure 1a, represents the simplest type of network and is often used in cellular automata models. Nodes are linked to their near neighbors only. Thus a regular lattice produces personal networks of limited size and a low whole network density. Because many of one node's neighbors will also be neighbors of each other,



(a) Regular Lattice: each node is linked to its four immediate neighbors.

(b) Random Network: most nodes have three or four links.

(c) Small World Network: most nodes are linked only to their immediate neighbors.

(d) Preferential Attachment (scale-free) network: a few nodes have many links.

Figure 1 *Examples Of Four Basic Models Of Networks With 30 Nodes. Generated using NetLogo (Wilensky, 2009).*

clustering will be high. But a regular lattice fails to meet the other criteria and is therefore a poor model of a social network.

Random linking, shown in Figure 1b, has been analyzed since the mid-twentieth century (Newman *et al.*, 2006: 12). Most nodes will have a similar number of links and the degree of connectivity follows a Poisson distribution. Thus "it is extremely rare to find nodes that have a significantly more or fewer links than the average" (Barabàsi & Bonabeau, 2003). Path lengths are short (Pool & Kochen, 1978/9; Dorogovtsev & Mendes, 2003: 105). But social networks are not in general created by making random links: rather, homophily dominates. So it is hardly surprising that random networks fail to replicate other key features of social networks. Indeed, the assortativity index of a random graph can be shown analytically to tend towards zero as the size of the network increases (Newman, 2002). So random networks are not good models of social networks either.

The 'small world' model, shown in Figure 1c, is produced by a few random re-wirings of a regular lattice to produce a model with high clustering and short paths (Watts & Strogatz, 1998). In effect, the small world model inherits its clustering from the regular lattice and its short paths from the random model (Dorogovtsev & Mendes, 2003: 105). The small-world model is closer to a social network than the regular lattice or random networks: it has high clustering and short paths. But it does not produce communities, nor nodes with high degrees of connectivity, nor display positive assortativity. Watts himself said: "the small-world model is not in general expected to be a very good model of real networks, including social networks" (Newman *et al*., 2006: 292).

The preferential attachment, or scale-free network model, shown in Figure 1d, is created by new nodes tending to link to those that already have many links (Barabàsi & Albert, 1999). This creates a hub-and-spoke pattern: many nodes have only one link and a few nodes have many links. The degree of connectivity follows a power law distribution, i.e., it is very highly right-skewed. The preferential attachment model can be criticized on the basis of its underlying dynamics. People do not usually know who has many links and even if they did would not necessarily want to link to these popular people, or the 'target' may not want to reciprocate. For instance, the failure of Milgram's and subsequent small world experiments, discussed above, could be taken as evidence that people have only limited information about others' connections. As with the random network, the assortativity index of the preferential attachment model can be shown analytically to tend to zero (Newman, 2002). But the preferential attachment model can produce low whole network density, a fat-tailed cumulative degree of connectivity, communities, and short paths.

Table 1 summarizes how the four basic network models score against the desirable characteristics just described. It suggests that none of them is a very good model of real personal and social networks.

A New Model

Our alternative model aims to reproduce all these basic features of large social networks. It is founded on the ideas of social space and social distance that can be traced back to Park (1924) and were developed by Heider (1958: 191) among others. The setting for the model is what could be called a social map. While a geographical map shows how places are distributed and linked, the social map does the same for people. In this model, the closer any pair of agents are, the shorter the social distance between them. If it were considered that geographical distance alone determined social relationships, then this social map

Characteristic	Regular	Random	Small-World	Preferential Attachment
Personal networks				
Size limited	✓	✓	✓	X
Size varies with right skewed distribution	X	X	X	✓
High clustering	✓	X	✓	X
Change over time	X	X	X	Only growth
Social networks				
Low density	✓	✓	✓	✓
Short path lengths	X	✓	✓	Possible
Positive assortativity	X	X	X	X
Communities	X	X	X	✓

Table 1 *Summary Of Characteristics Of The Four Basic Network Models.*

could become a geographical map with distance measured in miles or travel time. McFarland & Brown (1973: 226-7) suggested that social distance could be used in two distinct ways: to measure the strength of ties, where those who are short distances apart are more likely to interact; and to measure similarity, where short distances imply similar characteristics. In this model, social distance is used to indicate strength of tie.

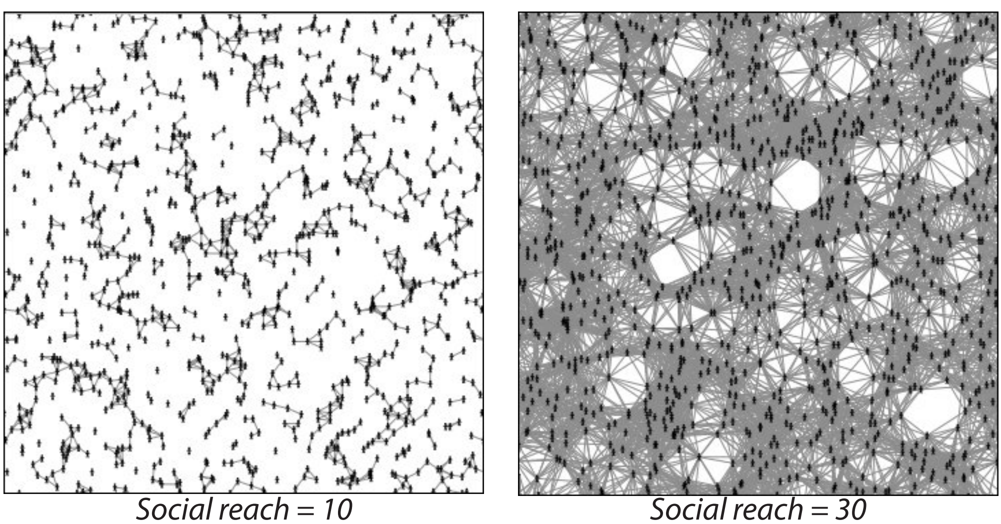
The model is based on the concept of social circles, an idea dating back to at least Simmel (1902). The term "circle" was then used as metaphor. Yet a circle has a very useful property in this context: the formal definition of a circle is "the set of points equidistant from a given point", the centre (Weisstein, 1998: 246). The circumference of a circle will contain all those points within a distance set by the radius—which will henceforth be called the 'social reach'—and creates a cut-off, limiting the size of personal networks. For a given distribution of agents in a social space, a small reach can create a disconnected society; a large social reach, a connected society. Alternatively, if the social reach is very small, it can be said to replicate a network of close family and friends: if bigger, it becomes a model for larger networks including acquaintances. Models similar to that proposed have been reported in the physics literature, e.g., Barthélemy (2003) and Hermann *et al.* (2003).

Agents are permitted to link only with agents who can reciprocate, i.e., agents must be within each other's social reach to link. If this were not so, then if A were to have a bigger social reach than B, B could be in A's circle but not vice-versa; this would imply that A 'knows' B but B does not 'know' A. Although there may be all sorts of asymmetries in the relationship between A and B, they must in some sense both 'know' each other. The simplest way to achieve reciprocity is for all agents to have the same reach, but this assumption is not essential, and will be relaxed later.

The properties of the social circle model will be illustrated with a series of simulations. These use a population of 1,000 agents, meaning that there are almost half a million possible undirected links (1,000 × 999 / 2). These agents are randomly distributed across an unbounded grid of just under 100,000 cells, thus producing a population density of about one percent. The results are averages of 30 runs.

Figure 2 illustrates two examples of the networks that can be created: the black dots indicate agents and the grey lines, the links between them. In both cases, communities (i.e., groups of agents that are well connected within themselves but loosely connected to other groups) can be seen. The connectedness of the networks can be measured by the extent to which adding "friends-of-friends" would increase the size of agents' personal networks (Grannis, 2010); the higher this "Grannis factor", the greater the connectedness. More importantly, if the Grannis factor is less than one, society comprises tiny groups which do not interact; if it is greater than one, then society is highly interconnected, or in network jargon, there is a giant component. With a social reach of ten, as illustrated in the left hand panel, the Grannis factor is typically 0.95; while with a reach of 30, illustrated in the right hand panel, it is 2.35.

Table 2 and Figure 3 summarize the results for social reaches ranging from 15 to 40. (To explore larger reaches, with larger personal networks, more agents would be needed in a larger world.) As the social reach is increased, the average personal network sizes increase. For a social reach of up to about 30, personal network sizes follow a Poisson distribution (where the mean is the same as the



Social reach = 10 *Social reach = 30*

Figure 2 *Samples Of Networks With Different Social Reaches. (Black nodes, grey links)*

Reach	15	20	30	40
Degree of connectivity (Personal network size)				
Average	7	13	28	51
Variance	*7*	*13*	*28*	*45*
Skewness	0.83	0.83	0.75	0.69
Cluster coefficient (average)	0.584	0.590	0.586	0.587
Density	0.007	0.013	0.028	0.051
"Grannis factor"	1.53	1.91	2.35	2.55
Assortativity				
Average	0.79	0.82	0.84	0.80
Pairwise	0.59	0.58	0.58	0.53

Table 2 *Summary Of Results For Various Fixed Reaches.*

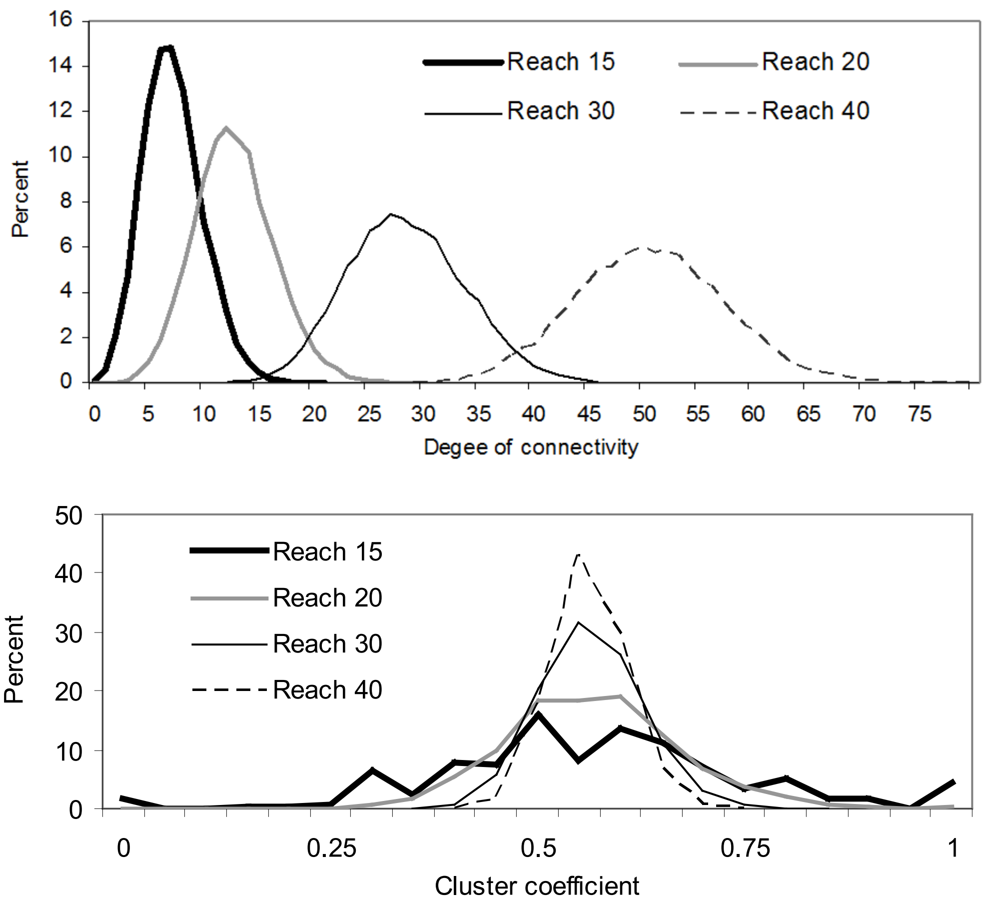


Figure 3 *Distributions Of Degree Of Connectivity And Clustering Coefficients Produced By Different Social Reaches. (Details in Table 2.)*

variance); but for larger social reaches, the mean tends to exceed the variance. The skewness of the distributions declines as the reach is increased. Of course, as the reach is increased, the network density increases too.

The clustering coefficient measures the extent to which those connected to an agent are in turn linked to each other (Scott, 1991: 74). When all agents have the same social reach, and where agents are located very close to each other, their circles will largely overlap and they will know most of the same agents. At the other extreme, when an agent is located on the circumference of another's circle, the geometry of circles implies that if agents are uniformly distributed over the space, then the minimum clustering coefficient will be 0.39. Furthermore, any agent will have a clustering coefficient of 0.56 or more with half of the agents in its network; and a coefficient of between 0.39 and 0.56 with the remainder. However, if the social reach is set very low, and thus the personal network size is small, then none of those in an agent's personal network may know each other, giving a clustering of zero; alternatively, all the agents may be close and know each other producing a clustering coefficient of one. Thus, as shown in Figure 3, the clustering coefficient will vary more for smaller social reaches than larger ones; and as the social reach increases, the minimum of the clustering coefficient will tend to 0.39 and the mean to 0.56.

Intuition suggests that this model should produce networks with positive assortativity by degree of connectivity because agents in densely populated regions will tend to have many links, as will those to whom they are linked (as Hermann *et al*., 2003, suggested). There is no clear agreement on how assortativity should be calculated (Newman, 2003; Newman *et al*., 2006: 555). Two measures are reported here: the correlation between an agent's degree of connectivity and the average degree of connectivity of those to which it is linked, and the correlation between the degree of connectivity between each pair of linked agents. Both measures show positive assortativity.

The minimum number of links from one agent to another, the path length, is determined by the size of the 'world', the social reach and the distribution of agents. Using geometry, it is possible to calculate the theoretical path length given the social reach and the size of the world. According to this calculation if the social reach is set at 30, the *maximum* number of steps between any pair of agents will be 7.4, given a world size of about 100,000 cells. But many agents will be closer, resulting in shorter path lengths. Simulations suggest that the average path length for 1,000 agents with a reach of 30 in a world of 100,000 cells is between five and six. Thus by choosing appropriate values of social reach and world size, this model can be used to reproduce social worlds consistent with Milgram's six degrees of separation.

Extending The Model

The simple uniform-reach model is inflexible in that the only parameters are population density and the size of the social reach. Furthermore, although it has positive assortativity, it does not produce very skewed distributions of connectivity. Also with larger social reaches all agents will have a clustering coefficient of at least 39 percent.

The simplest way to increase the flexibility of the model is to split the population of agents into two or more groups and allocate each group a different reach. Going further, it is possible to allow each agent to have a different reach. Rather than choosing the percentages of agents with given social reaches, it now becomes necessary to choose the distribution of social reaches and the parameters of those distributions. There is not an obvious choice. (But, as before, links are only permitted between pairs that can reciprocate. For example, if agent A has a social reach of 25 and agent B has a reach of 30, then providing the distance between A and B is no more than 25, they can link.)

For illustration, five sets of parameters were used to produce personal networks with an average degree of about 12, representing a typical number of stronger ties as noted earlier. The first is one where all agents have the same reach. The second uses two reaches; in this case, a quarter of agents having a larger reach. For the others, the agents' reaches are determined by one of three distributions:

- A Poisson distribution, for which the only parameter is the mean (which by definition equals the variance);
- A power distribution, generated by taking the exponential of a gamma function. This requires the mean and variance of the gamma distribution; and to avoid very small and very high reaches, minimum and maximum reaches must be set, and;
- A uniform distribution, where the minimum and maximum reaches are the parameters.

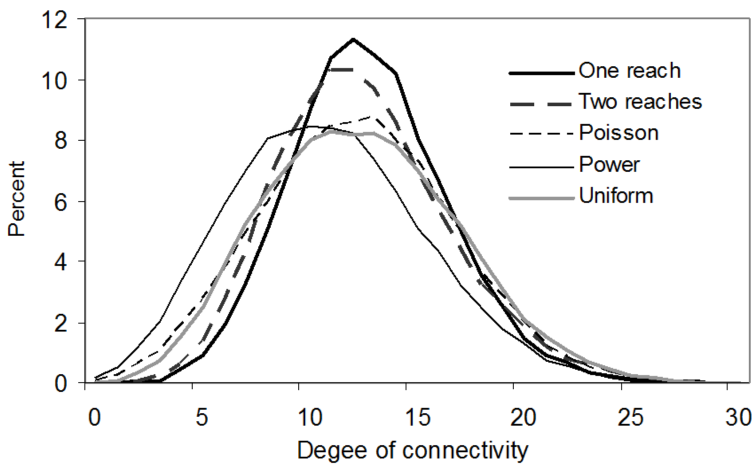
The results are shown in Table 3 and Figure 4. All produce distributions of degrees of connectivity—personal network sizes—that are positively skewed, the skew of the two-reach model being particularly strong. Compared with using a single fixed reach, the alternatives reduce the strength of positive assortativity, but increase the ranges of the size of both personal networks and the clustering coefficients.

Adding Dynamics

It was noted that personal networks were constantly changing. Putting aside the issue of changes due to fertility and mortality, which can be added if appropriate, personal networks change because people drift apart, either by physically moving away or by changing behavior. To accommodate this, a change pa-

Type of distribution	One reach	Two reaches	Poisson	Power	Uniform
Parameters for reach	20	Smaller = 19 Larger = 30 % larger = 25	Mean = 22	Min = 15 Max = 30 Gamma: mean = 20 var = 1	Min = 15 Max = 30
Degree of connectivity (Personal network size)					
Average	12.7	12.5	12.2	12.2	12.4
Variance	*13.1*	*16.2*	*20.0*	*19.9*	*20.2*
Skewness	0.83	1.11	0.61	0.48	0.47
Cluster coefficient (average)	0.590	0.578	0.595	0.595	0.594
Density	0.013	0.012	0.012	0.012	0.012
"Grannis factor"	1.91	2.18	2.21	2.22	2.25
Assortativity					
Average	0.82	0.75	0.65	0.66	0.66
Pairwise	0.58	0.46	0.40	0.39	0.39

Table 3 *Ways Of Producing An Average Personal Network Of Size About 12.*



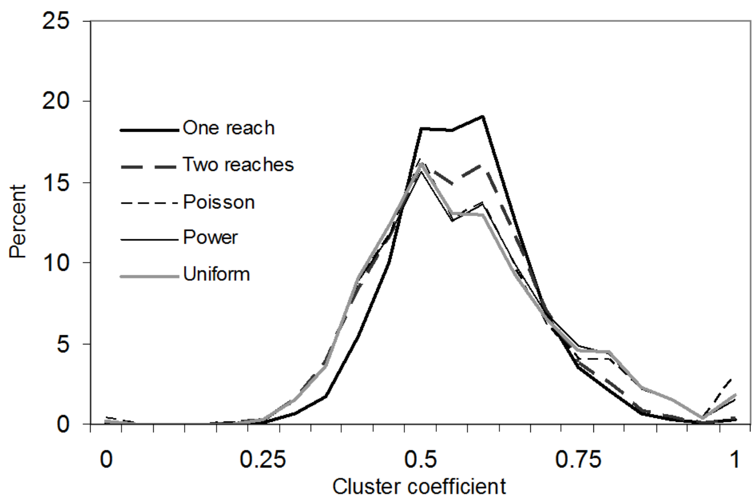


Figure 4 *Distributions Of Degree Of Connectivity And Clustering Coefficients Produced By Different Methods Of Generating A Personal Network With A Size Averaging About 12. (Details in Table 3.)*

rameter, called 'social shift', can be introduced that allows a proportion of agents to move in the social space. If this change is random, it will change the size and composition of individual agents' networks, but it will not change the overall structure of the society.

When an agent moves, its personal network may change in size, composition or both. Furthermore, moves may affect others who do not themselves move. The larger the social reach, the more agents will potentially be affected by another agent's move. The effect of this social shifting on agents' personal networks was examined, according to whether or not they moved and whether or not the size or composition of their personal networks changed. If neither the number nor identity of agents in their personal network changed, they were counted as unaffected. Simulation showed that if just 1 in 20 agents shift one step each period, then over ten periods, a third of small personal networks and almost two-thirds of larger ones will change; over 25 time periods, between 60 and 90 percent will change, depending on the network sizes.

Discussion And Summary

The four standard network models (regular lattice, random, small-world and preferential attachment) do not reproduce well the characteristics of social networks. A simple method to create large social networks in agent-based models has been presented, drawing on the metaphor of social circles and making use of the geometrical properties of circles. The method meets all the desired criteria in that it creates personal networks that:

- Are limited in size by using the social reach as a cut-off;
- Vary in size between individuals by randomly distributing agents across the social map and, further, by varying the size of the social reach; and can have right-skewed distributions of connectivity;
- Display high clustering, generated by the overlapping social reaches: the clustering coefficient tends to average around 0.5, but can vary from zero to one for individual agents depending on the parameters chosen;
- Can change over time. By allowing agents to move randomly, changes can be made in personal networks while maintaining the overall network structure. The cumulative effect over a long period of time of a small proportion of agents moving a small distance is that the size and identity of the personal networks of almost all the agents change.

It also produces social networks that:

- Have low whole network density: the smaller the social reach, the lower the whole network density;
- Are positively assortative by degree of connectivity, i.e., well-connected agents tend to be connected to other well-connected agents;

- Have communities, and;
- Can have short path lengths depending on the values chosen for social reach, number of agents and world size.

There are two key assumptions underlying the model: symmetrical relationships and the use of two-dimensional social space. Overall, this simple method can be used to create agent-based models that represent empirical social networks with greater veracity than the four standard network models.

Acknowledgment

The initial work was supported by Microsoft Research through its European PhD Scholarship Programme.

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